CORRESPONDENCE

Comments on "Numerical Advection Experiments"

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It was interesting to note of the fourth-order methods analyzed by Crowley [1] that the nonconservative scheme had somewhat better phase and amplitude characteristics than the conservative scheme. The simplicity of the nonconservative scheme over that of the conservative form suggested that if the former could be written in conservative form it would be doubly advantageous to do so. It will be demonstrated here that this can be done and further it will be shown that the method can be programmed to be faster than the usual methods.

In Crowley's nonconservative scheme

$$\psi_j^{N+1}=[(I-A)\psi^N]_j \tag{1}$$

where

$$(I\psi)_j=\psi_j \tag{2}$$

and

$$\begin{aligned}(A\psi)_j=&\frac{\alpha}{12}[8(\psi_{j+1}-\psi_{j-1})-(\psi_{j+2}-\psi_{j-2})]\\&+\frac{\alpha^2}{24}[30\psi_j-16(\psi_{j+1}+\psi_{j-1})+(\psi_{j+2}+\psi_{j-2})]\\&+\frac{\alpha^3}{12}[-2(\psi_{j+1}-\psi_{j-1})+(\psi_{j+2}-\psi_{j-2})]\\&-\frac{\alpha^4}{24}[6\psi_j-4(\psi_{j+1}+\psi_{j-1})+(\psi_{j+2}+\psi_{j-2})].\end{aligned} \tag{3}$$

We may write a two-step flux form of (3) as

$$\frac{\Delta t}{\Delta x}F^{(1)}_{j-1/2}=\frac{2}{3}\alpha_{j-1/2}\left(1-\frac{\alpha^2_{j-1/2}}{4}\right)[(\psi_{j-1}+\psi_j)+\alpha_{j-1/2}(\psi_{j-1}-\psi_j)] \tag{4}$$

$$\frac{\Delta t}{\Delta x}F^{(2)}_j=\frac{-\alpha_j}{12}(1-\alpha_j^2)[(\psi_{j-1}+\psi_{j+1})+\frac{\alpha_j}{2}(\psi_{j-1}-\psi_{j+1})]. \tag{5}$$

If we now write

$$\psi_j^{N+1}=\psi_j^N+\frac{\Delta t}{\Delta x}(F^{(1)}_{j-1/2}+F^{(2)}_{j-1})-\frac{\Delta t}{\Delta x}(F^{(1)}_{j+1/2}+F^{(2)}_{j+1}). \tag{6}$$

With $\alpha_{j-1/2}$, α_{j-1}, $\alpha_{j+1/2}$ and α_{j+1} set equal to α, (6) is equivalent to (1), (2), and (3).

Of particular interest is that both (4) and (5) may be evaluated without the complexity that arises from incorporating additional points of the mesh as normally becomes necessary with a fourth-order approximation. The procedure may best be implemented in obtaining ψ_j^{N+1} by incrementing ψ_j^N successively with flux contributions from the left and then from the right, where the latter simultaneously gives the flux from the left for ψ_{j+1}^{N+1}. Equation (5) is evaluated as a separate entity so that it may be centered at index j.

It is noteworthy that in incrementing the values to ultimately obtain the ψ^{N+1} there is no repetition of any part of the calculation. This is in contrast to the direct extension of the nonconservative form to the conservative form. It should also be emphasized that while a similar formulation could be applied to Crowley's fourth-order conservative scheme the difficulty of spanning three cells in the second step would complicate matters considerably.

REFERENCE

1. W. P. Crowley, "Numerical Advection Experiments," *Monthly Weather Review*, vol. 96, No. 1, Jan. 1968, pp. 1–11.

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Reply *

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Dr. Fromm has produced an intriguing generalization of the nonconservative advection formulation. For the special case of a constant velocity field, his equation (6) is, as he points out, an approximation to the color equation,

$$\frac{\partial\psi}{\partial t}+u\frac{\partial\psi}{\partial x}=0.$$

For an arbitrary velocity field however, his equation (6) is an approximation to the conservation equation,

$$\frac{\partial\psi}{\partial t}+\frac{\partial\psi u}{\partial x}=0$$

where an estimate of the flux, ψu, is provided by the quantities $F^{(1)}_{j-1/2}$ and $F^{(2)}_j$ in his equations (4) and (5).

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PICTURE OF THE MONTH

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A late spring storm which traversed southeastern Canada between June 2 and June 4, 1968, brought a pool of cold air into the northeastern United States. An interesting feature of this storm was the development of postfrontal thunderstorms along a line poleward and roughly parallel to the jet stream. This line of activity moved eastward and produced weather more severe than that which accompanied the initial cold front passage.

Figure 1 shows the ESSA 5 view of the storm at 2058 GMT on June 2. The accompanying surface analysis and jet stream position appear in figure 2. At this time the Low is centered south of Hudson Bay (78°W., 49°N.). The frontal cloud band (A, B) (fig. 1) appears broad and solid over the New England States and less organized over the Mid-Atlantic States. Numerous shadows, cast by towering cumulonimbi embedded in the frontal band, can be seen north and east of New York (C). A large relatively clear area appears along southern Indiana and Ohio and western New York. Upper air observations indicate that the jet stream lies within this clear area (fig. 2).

Forecasts prepared 12 hr. before the picture indicate no significant weather for the lower Great Lakes region, and early morning ESSA 2 and 6 passes showed clear

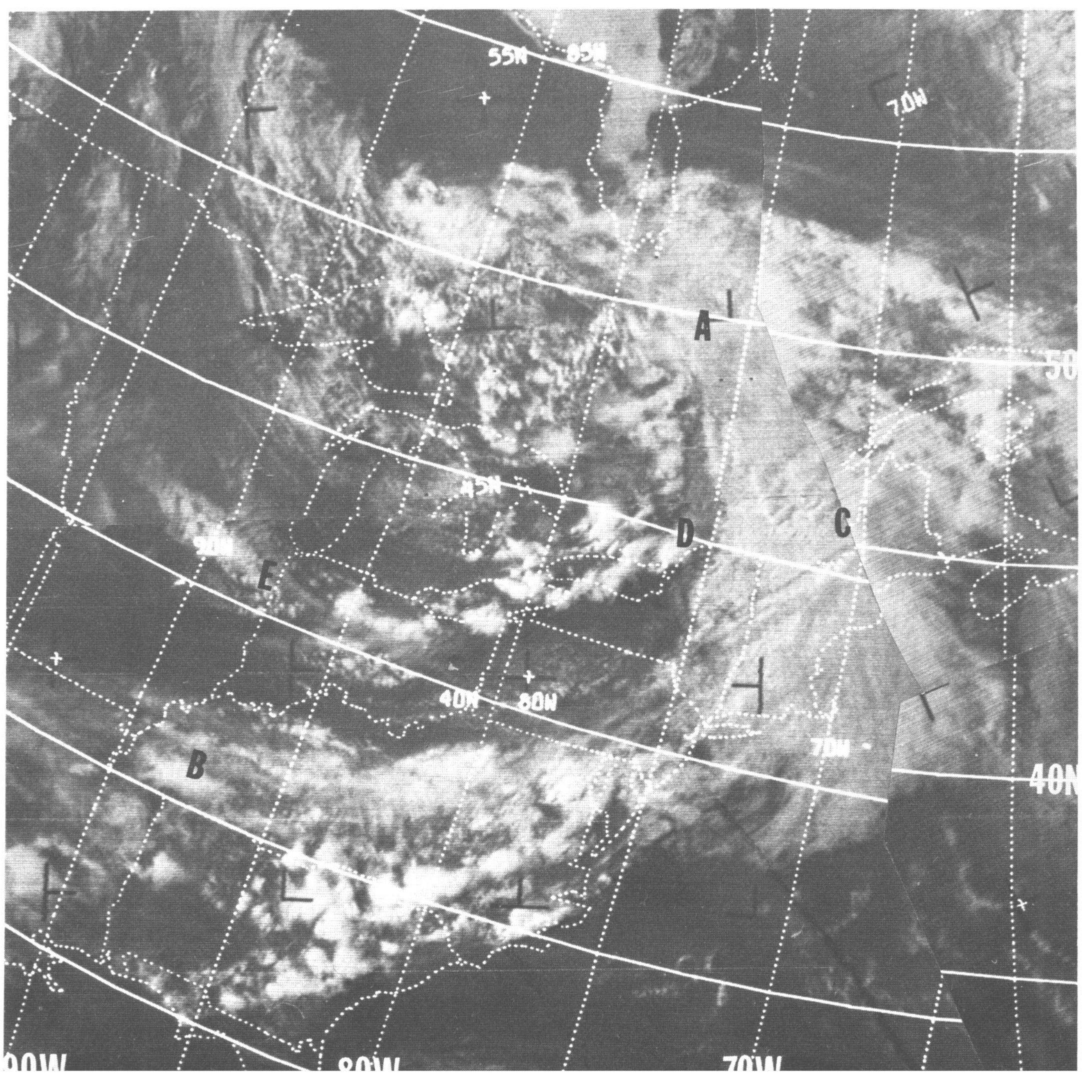

FIGURE 1.—ESSA 6 Pass 5188-9, 1905–2058 GMT, June 2, 1968.

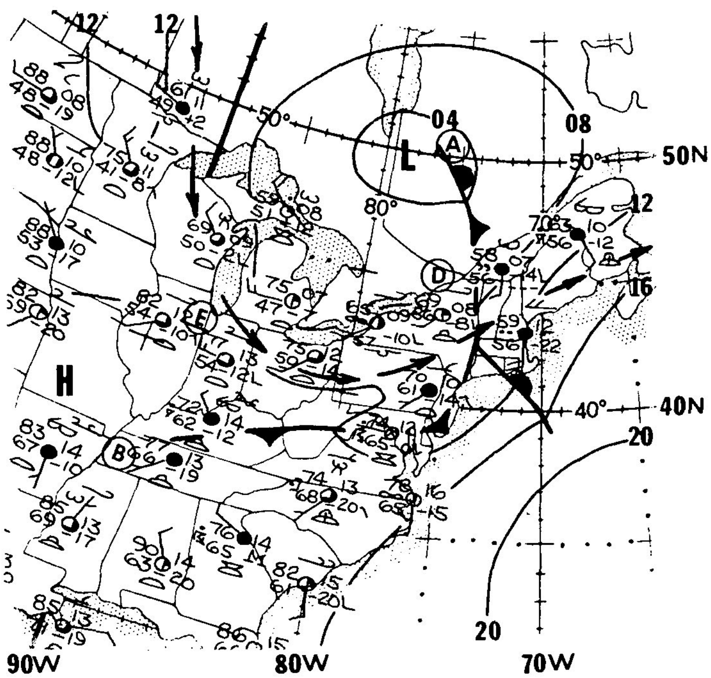

FIGURE 2.—Surface analysis 2100 GMT, June 2, 1968, and jet stream position, 0000 GMT, June 3, 1968.

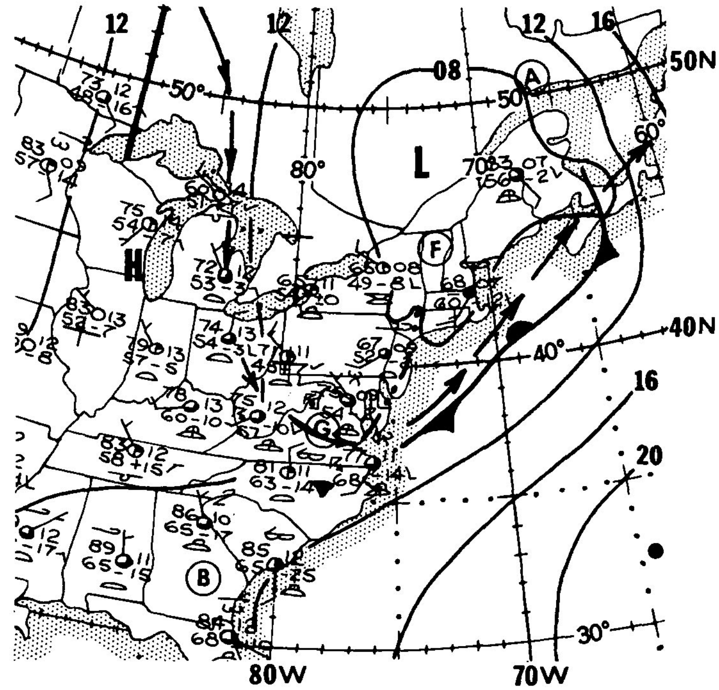

FIGURE 4.—Surface analysis 2100 GMT, June 3, 1968, and jet stream position, 0000 GMT, June 4, 1968.

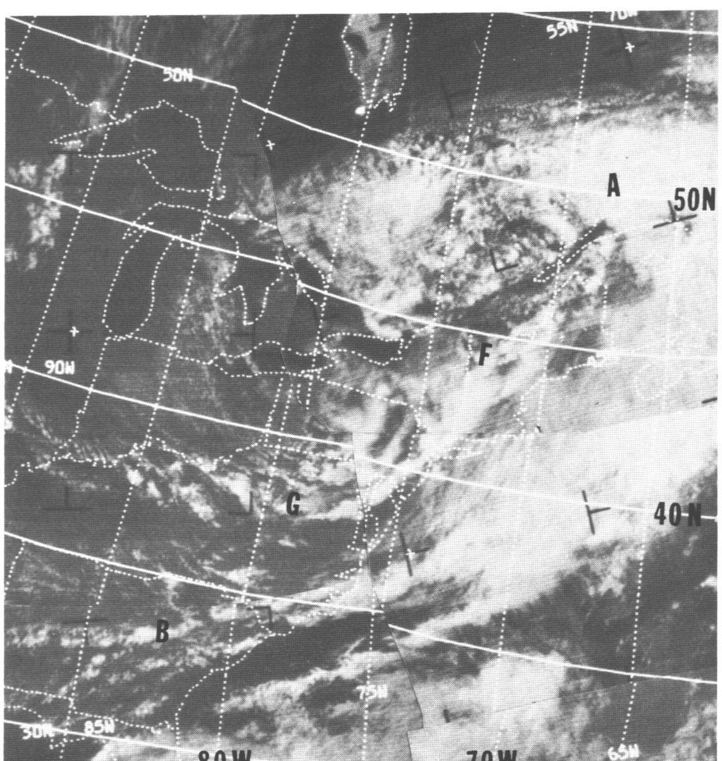

FIGURE 3.—ESSA 6 Pass 5201–2, 1929–2102 GMT, June 3, 1968.

sky conditions from Kentucky to Grand Rapids, Mich. Late morning and early afternoon heating of the cold, relatively unstable air behind the jet stream produced large cumulonimbus clusters which appear in figure 1 along a line extending from Lakes Ontario and Erie westward to Illinois (D, E). Precipitation reports indicate a trace to 0.25 in. of rainfall in this region. This area of activity gradually moved southeastward during the night and the early morning APT passes on June 3 showed the remnants of it in western Pennsylvania.

By 1900 GMT on June 3, the low pressure area in Canada had moved 5° eastward and most of the frontal cloudiness was now off the United States mainland (A, B, fig. 3). Behind the front, a solid line of thunderstorms (F, G) can be seen extending from New England southward into Virginia. At this time heavy thunderstorms and golf ball-size hail was being reported in Scranton, Pa., and the Washington metropolitan area.

The line of post-frontal showers maintained the same relationship with the jet core on both days. It is believed that the deep unstable air poleward of the jet core as well as the vertical shear along the jet were the factors responsible for the formation and persistence of this activity. A slight increase in the amplitude of the upper level trough along with the southward shift of the jet stream (fig. 4) may also have contributed to the development and intensity of the cold air thunderstorms.

The developing thunderstorm line could easily be seen in the satellite pictures on June 2, but it was difficult to identify it in the stability or moisture analysis which is based on upper air observations alone. Satellite pictures can provide the local forecaster with much useful information for the diagnosis and prediction of small-scale weather systems such as this.